



Estimate of SU(3) flavour symmetry breaking in $\bar{B}_s \rightarrow K^+ K^-$ vs. $\bar{B}_d \rightarrow \pi^+ \pi^-$ decay

M Beneke

Institut für Theoretische Physik E, RWTH Aachen, Sommerfeldstr. 28, D - 52074 Aachen

I estimate the SU(3) flavour symmetry breaking in the ratio of penguin-to-tree ratios of the decays $\bar{B}_s \rightarrow K^+ K^-$ and $\bar{B}_d \rightarrow \pi^+ \pi^-$, given an assumption on the flavour symmetry breaking of the hadronic input parameters. The decay amplitudes are calculated in QCD factorization. Implications for the determination of γ are discussed.

1 Introduction

In this note I perform a study of SU(3) breaking for the decays $\bar{B}_d \rightarrow \pi^+ \pi^-$ and $\bar{B}_s \rightarrow K^+ K^-$. This system is interesting, because it allows for a determination of the angle γ from mixing-induced and direct CP asymmetries [1, 2] provided the SU(3) symmetry breaking corrections to a certain double ratio of amplitudes are known. The analysis of SU(3) breaking is done in the theoretical framework of QCD factorization [3, 4, 5], which expresses the hadronic decay amplitudes in terms of fundamental constants, decay constants, form factors etc. in the heavy quark limit. The following is a preliminary version of work in progress in collaboration with M. Neubert.

We write the decay amplitudes as

$$\begin{aligned} A(\bar{B}_d \rightarrow \pi^+ \pi^-) &= \frac{i G_F}{\sqrt{2}} \left(\lambda_u^{(d)} T_\pi + \lambda_c^{(d)} P_\pi \right), \\ A(\bar{B}_s \rightarrow K^+ K^-) &= \frac{i G_F}{\sqrt{2}} \left(\lambda_u^{(s)} T_K + \lambda_c^{(s)} P_K \right) \end{aligned} \quad (1)$$

where $\lambda_p^{(D)} = V_{pb} V_{pD}^*$. The “tree” and “penguin” amplitudes are defined as the coefficients of the two terms with different CKM matrix elements. They also contain sub-leading penguin, electroweak penguin and weak annihilation amplitudes. In the SU(3) symmetry limit $T_\pi = T_K$ and $P_\pi = P_K$. We will be interested in the deviations of the ratios

$$r_T \equiv \frac{T_K}{T_\pi}, \quad r_{PT} \equiv \frac{P_K/T_K}{P_\pi/T_\pi} \quad (2)$$

from unity.

The ratio of tree amplitudes is the product of a factorizable and a non-factorizable term, $r_T \equiv r^f r_T^{\text{nf}}$. The factorizable term is given by

$$\begin{aligned} r^f &= \frac{(1 - m_K^2/m_{B_s}^2)(1 - 4m_K^2/m_{B_s}^2)^{1/2}}{(1 - m_\pi^2/m_{B_d}^2)(1 - 4m_\pi^2/m_{B_d}^2)^{1/2}} \frac{f_K}{f_\pi} \frac{F_0^{B_s \rightarrow K}(m_K^2)}{F_0^{B_d \rightarrow \pi}(m_\pi^2)} \\ &= 1.19 \times \frac{F_0^{B_s \rightarrow K}(m_K^2)}{F_0^{B_d \rightarrow \pi}(m_\pi^2)} \approx 1.27, \end{aligned} \quad (3)$$

where we have included the light meson masses and phase space effects. The $B \rightarrow \pi$ form factor is taken from [6], but the assumed value for the $B_s \rightarrow K$ form factor is only an “educated guess”. In the context of QCD sum rules this form factor is expected to depend sensitively on the poorly known first Gegenbauer moment of the kaon light-cone distribution amplitude, since the form factor is dominated by the soft spectator-quark overlap term.

2 Estimate of SU(3) breaking

The factorizable SU(3) breaking correction cancels in the double ratio r_{PT} [2]. This ratio and r_T^{nf} deviate from unity due to non-factorizable effects that can be computed in QCD factorization assuming that $m_b \gg \Lambda_{\text{QCD}}$. The computation is a straightforward extension of the results of [4]. I refer to this paper and [5] for all details concerning the method and the values of the hadronic parameters. The parameters relevant to SU(3) breaking are:

- r_χ^K vs. r_χ^π in the normalization of scalar penguin terms;
- the ratio $f_{B_s} f_K / (m_{B_s}^2 F_0^{B_s \rightarrow K})$ vs. $f_B f_\pi / (m_B^2 F_0^{B \rightarrow \pi})$ in the normalization of the hard-scattering and weak annihilation terms;
- the Gegenbauer moments α_1^K, α_2^K vs. $\alpha_1^\pi = 0, \alpha_2^\pi$, and the first inverse moments of the B -meson distribution amplitudes, m_B/λ_B vs. m_{B_s}/λ_{B_s} ;
- the parameters for soft power corrections from hard scattering and weak annihilation, X_A and X_H .

Since these parameters are often not well-known, we will make assumptions on the SU(3) breaking and exhibit the effect of these assumptions on the ratios r_T^{nf} and r_{PT} . This should be distinguished from more ambitious approaches as discussed in [7], where the SU(3) breaking in the input parameters is also computed.

For a given pair of related parameters denoted (x_π, x_K) , for instance $(x_\pi, x_K) = (\lambda_B, \lambda_{B_s})$, we set all other parameters and x_π to their standard values. We then write $x_K =$

$x_\pi(1 + \delta)$ and vary δ , which controls the amount of SU(3) breaking, between -0.3 and 0.3 . For the complex parameters X_H and X_A we vary the magnitude and phase simultaneously and independently by this amount. We make two exceptions to this treatment. For the light meson Gegenbauer moments, which are already small corrections to the SU(3) symmetric asymptotic distribution amplitudes, we take $\alpha_1^K = 0.2 \pm 0.2$, $\alpha_2^K = 0.1 \pm 0.3$, and $\alpha_2^\pi = 0.1$. Second, we use

$$\frac{r_\chi^K}{r_\chi^\pi} = \frac{2m_K^2}{m_\pi^2} \frac{1}{m_s/m_q + 1} = 0.99 \pm 0.06 \quad (4)$$

with $m_s/m_q = 24.2 \pm 1.5$ taken from [8].

The resulting variations of the magnitudes and phases of r_T^{nf} and r_{PT} about their default values

$$r_T^{\text{nf}} = 0.99 e^{0.2^\circ i}, \quad r_{PT} = 1.02 e^{-1.1^\circ i} \quad (5)$$

are shown in the upper part of Table 1. Adding up the various uncertainties we obtain non-factorizable SU(3) breaking effects of order $\pm 5\%$ for the ratio of tree amplitudes and of order $\pm 10\%$ for the double ratio of penguin and tree amplitudes. Since the strong phases of the ratios are very small, the SU(3) breaking effect on the phases is also small in absolute magnitude.

These estimates have to be regarded with caution, since they rely on the default estimate of weak annihilation ($\mathcal{Q}_A = 0$), in which the annihilation amplitude does not have a strong phase. To study the impact of weak annihilation in more detail we pick the four values X_A^i corresponding to $|\mathcal{Q}_A| = 0.75$ with phase 0, 90, 180, 270 degrees for the pion mode and allow the corresponding parameters to vary by $\pm 30\%$ for the kaon mode. All the values of the annihilation parameter chosen by this procedure lie (almost) within our standard error assumption $|\mathcal{Q}_A| < 1$. The lower part of Table 1 demonstrates that the SU(3) breaking effect on the ratio of penguin amplitudes can be large, up to 30% in magnitude and $\pm 15^\circ$ on the phase of r_{PT} . We conclude that unless a better understanding of SU(3) breaking effects in the weak annihilation amplitude is found the assumption $r_{PT} = 1$ should not be made.

3 Determination of γ

To estimate the theoretical uncertainty in the determination of the angle γ due to the SU(3) breaking effects, we assume that the mixing phase $\phi_{B_d} = 2\beta$ is 47° , corresponding to $\sin(2\beta) = 0.731$, and define the time-dependent CP asymmetry through

$$\begin{aligned} A_{\text{CP}}(t) &= \frac{\text{Br}(B(t) \rightarrow f) - \text{Br}(\bar{B}(t) \rightarrow \bar{f})}{\text{Br}(B(t) \rightarrow f) + \text{Br}(\bar{B}(t) \rightarrow \bar{f})} \\ &= -S_f \sin(\Delta m_B t) + C_f \cos(\Delta m_B t). \end{aligned} \quad (6)$$

Table 1. SU(3) breaking effect on the magnitude and phase of r_T^{nf} and r_{PT} for variations of a given input parameter as described in the text.

Parameter	$\delta r_T^{\text{nf}} $	$\delta \arg(r_T^{\text{nf}})$	$\delta r_{PT} $	$\delta \arg(r_{PT})$
r_χ	± 0.01	$< 0.1^\circ$	± 0.05	$< 0.1^\circ$
$F_0^{B \rightarrow M}$	$+0.02$ -0.03	$< 0.1^\circ$	$+0.05$ -0.06	$< 0.5^\circ$
λ_B	$+0.02$ -0.04	$< 0.3^\circ$	$+0.03$ -0.02	$< 0.1^\circ$
α_1^M	± 0.01	$< 0.1^\circ$	± 0.02	$\pm 1^\circ$
α_2^M	$+0.03$ -0.04	$\pm 0.2^\circ$	$+0.01$ -0.00	$< 0.1^\circ$
X_H	± 0.01	$< 0.1^\circ$	± 0.01	$< 0.1^\circ$
$X_A (\mathcal{Q}_A^\pi = 0)$	$+0.00$ -0.01	$< 0.1^\circ$	$+0.10$ -0.07	$\pm 0.5^\circ$
Sum	$+0.04$ -0.06	$\pm 0.2^\circ$	$+0.13$ -0.10	$\pm 1.2^\circ$
X_A^1	± 0.00	$< 0.1^\circ$	$+0.29$ -0.13	$\pm 1^\circ$
X_A^2	± 0.01	$< 0.5^\circ$	$+0.19$ -0.10	$+13^\circ$ -5°
X_A^3	± 0.00	$< 0.1^\circ$	$+0.00$ -0.03	$< 0.5^\circ$
X_A^4	± 0.01	$< 0.5^\circ$	$+0.18$ -0.09	$+5^\circ$ -14°

We assume that $S_{\pi\pi} = -0.6$ and $C_{\pi\pi} = -0.1$ have been measured. We can then determine

$$de^{i\theta} \equiv \left| \frac{\lambda_c^{(d)}}{\lambda_u^{(d)}} \right| \frac{[-P_\pi]}{T_\pi} \quad (7)$$

as a function of γ ,¹ and predict C_{KK} , the direct CP asymmetry in $B_s \rightarrow K^+ K^-$ decay, for a given assumed value of the SU(3) breaking ratio r_{PT} . A measurement of C_{KK} then results in a determination of γ with a theoretical error due to the uncertainty of r_{PT} .

We dismiss solutions with $d > 1$ as unphysical, since a large penguin amplitude is or will be excluded by branching fraction measurements. We generically find two solutions for $de^{i\theta}$. The second solution satisfies $d < 1$ only in a small range around $\gamma = 170^\circ$ and will not be discussed further. The other solution already constrains $5^\circ < \gamma < 95^\circ$ from $d < 1$ alone. The question is how a measurement of C_{KK} narrows this range further. In Figure 1 we show C_{KK} as a function of γ for five values of r_{PT} : the central value according to (5), and for the most pessimistic error assumptions corresponding to 1.3, 0.85, $e^{15^\circ i}$ and $e^{-15^\circ i}$. The figure also shows d and $\cos \theta$.

The CP asymmetry exhibits a resonance-like behaviour after d has gone through its minimal value, since the denominator of the expression for C_{KK} goes through a minimum when $d \cos \theta$ becomes small and positive. We find that the largest error is introduced by the uncertainty in the phase

¹The definition of $de^{i\theta}$ differs from [2] by a sign, so that θ is near zero in QCD factorization.

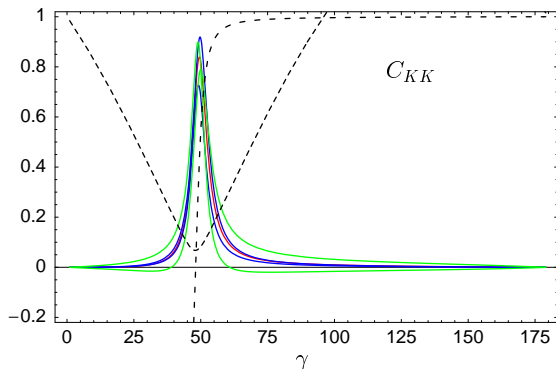


Figure 1. The direct CP asymmetry in $B_s \rightarrow K^+ K^-$ as a function of γ for five values of r_{PT} : $1.02 e^{-1^\circ i}$ (dark/red), 1.3, 0.85, (dark/blue) $e^{15^\circ i}$ and $e^{-15^\circ i}$ (light/green). The dashed lines show d and $\cos \theta$. $\cos \theta$ changes rapidly around $\gamma = 50^\circ$.

of r_{PT} , which in our analysis is entirely due to weak annihilation. For instance, if $C_{KK} = 0.06$ is found, the strategy would determine two values of γ in the absence of theoretical (and experimental) errors. Including the SU(3) breaking error, we obtain $\gamma = (40_{-6}^{+3})^\circ$ and $\gamma = (65_{-9}^{+13})^\circ$. Since $\cos \theta$ is expected to be positive, the second range is theoretically favored. In general, the theoretical error can become larger or smaller depending on C_{KK} as seen from the figure. It will also depend on what the mixing-induced and direct CP asymmetries in $B_d \rightarrow \pi^+ \pi^-$ will eventually turn out to be, but our result should be generic.

We therefore conclude that the strategy to determine γ from the $B_d \rightarrow \pi^+ \pi^-$ to $B_d \rightarrow K^+ K^-$ system *may* suffer from considerable theoretical uncertainties, unless additional information is available. This can come from two sources: (1) Excluding the possibility of a weak annihilation amplitude with a large strong rescattering phase would already eliminate the largest uncertainty that we could identify in the context of QCD factorization. This could be achieved experimentally by excluding a large direct CP asymmetry in $B \rightarrow \pi^- K^+$ and neglecting SU(3) breaking effects in the estimate of the weak annihilation phase. (2) If the $B_s \bar{B}_s$ mixing phase is known, the mixing-induced CP asymmetry in $B_s \rightarrow K^+ K^-$ decay provides a fourth observable, which could be used to determine (or eliminate) θ , leaving only the SU(3) breaking error on d . To make this work in practice, a significant experimental effort is required.

References

1. I. Dunietz, *Extracting CKM parameters from B decays*, FERMILAB-CONF-93-090-T, presented at the Summer Workshop on B Physics at Hadron Accelerators, Snowmass, Colorado, June 21 – July 2, 1993.
2. R. Fleischer, Phys. Lett. B **459** (1999) 306;

R. Fleischer and J. Matias, Phys. Rev. D **66** (2002) 054009.

3. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. B **591**, 313 (2000).
4. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606** (2001) 245.
5. M. Beneke and M. Neubert, [hep-ph/0308039].
6. P. Ball, talk at this Workshop [hep-ph/0306251].
7. A. Khodjamirian, talk at this Workshop.
8. H. Leutwyler, Phys. Lett. B **378** (1996) 313.